

<b>Optional Assignment</b>	Advanced Control Systems	5 <sup>th</sup> Sem. EE
Marks 0	Autumn-2018	

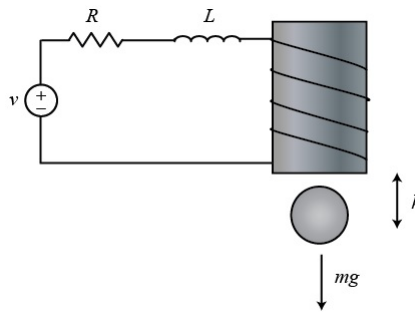
The assignment needs to be solved using MATLAB. Solved submissions can be submitted electronically by 20 Feb 2018. Any discussions required can be held between 16-20 Feb 2018.

The figure below shows a magnetically suspended ball. The current through the coils induces a magnetic force which can balance the force of gravity and cause the ball (which is made of a magnetic material) to be suspended in mid-air. The equations for the system are given by:

$$m \frac{d^2 h}{dt^2} = mg - \frac{K i^2}{h} \quad (1)$$

$$V = L \frac{di}{dt} + iR \quad (2)$$

where  $h$  is the vertical position of the ball,  $i$  is the current through the electromagnet,  $V$  is the applied voltage,  $m$  is the mass of the ball,  $g$  is the acceleration due to gravity,  $L$  is the inductance,  $R$  is the resistance, and  $K$  is a coefficient that determines the magnetic force exerted on the ball.



- The system is at equilibrium (the ball is suspended in mid-air) whenever  $h = Ki^2/mg$  (at which point  $dh/dt = 0$ ). Assuming the following values for various parameters:  $m = 0.05kg$ ,  $K = 0.0001$ ,  $L = 0.01H$ ,  $R = 1\Omega$ ,  $g = 9.81m/s^2$ , linearise the equations about the point  $h = 0.01$  m  $i = 7$  A and obtain the following linear state-space equations:

$$\frac{dx}{dt} = Ax + Bu \quad (3)$$

$$y = Cx + Du \quad (4)$$

where

$$x = \begin{bmatrix} \Delta h \\ \Delta \dot{h} \\ \Delta i \end{bmatrix} \quad (5)$$

$x$  is the set of state variables for the system,  $u$  is the deviation of the input voltage from its equilibrium value ( $u = \Delta V$ ), and  $y$  (the output) is the deviation of the height of the ball from its equilibrium position ( $y = \Delta h$ ).

- If step-1 is completed correctly, the following matrices to describe the state-space model would be obtained:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 980 & 0 & -2.8 \\ 0 & 0 & -100 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix}, C = [ 1 \ 0 \ 0 ] \quad (6)$$

Now analyse the linear system for stability and see how it responds to a non-zero initial condition given by

$$x_0 = \begin{bmatrix} 0.01 \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

Plot the ball position with respect to time.

- Test the system for controllability and observability and design a full state-feedback controller and place the closed-loop poles such that the settling time is less than 0.5 sec and overshoot is less than 5%. The third pole should be placed such that the response due to it is sufficiently fast and that it won't have much effect on the overall response. Plot the ball position with respect to time once the control has been implemented and verify that the design criteria has been met.

4. Design and add a feed-forward control block  $N$  to the controller such that it tracks a step input correctly. Again, show the result obtained using a plot.
5. Design a full order observer and an observer based controller for the system. Assume the same control problem as in Step-3. Assume a suitably fast error dynamics. Plot the error dynamics and the controlled response.