Department of Electrical Engineering
Islamic University of Science & Technology

Tutorial-I	Advanced Control Systems	5^{th} Sem. EE
Marks 10	Autumn-2017	

Questions can be discussed in class if required. Solved submissions would be accepted on 29^{th} *November 2017 till 10:00 AM only. Clarity and neatness of submission is important.*

1. Express $v = (3, 7, -4)$ in R^3 as a linear combination of the vectors $u_1 = (1, 2, 3)$, $u_2 = (2, 3, 7)$ and $u_3 = (3, 5, 6)$.

2. Do $u_1 = (1, 2, 3)$, $u_2 = (2, 3, 7)$ and $u_3 = (1, 5, 9)$ constitute a basis for R^3?

3. Can the polynomial $v = 3t^2+5t-5$ be expressed as a linear combination of the polynomials $p_1 = t^2+2t+1$, $p_2 = 2t^2+5t+4$ and $p_3 = t^2+3t+6$. If yes, how?

4. Is the space V consisting of all 2×2 matrices a vector space? If yes, verify and propose a basis for such a space. What is the dimension of V?

5. Suppose the vectors u, v, w are linearly independent. Show that the vectors $u+v, u-v, u-2v+w$ are also linearly independent.

6. W consisting of symmetric 2×2 real matrices is a vector space of dimension 3. Show that the following matrices are a basis for W.

$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}; E_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; E_3 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

7. A feedback system has a closed loop transfer function $\frac{50(1+s/5)}{s(1+s/2)(1+s/50)}$. Construct the first companion form, the second companion form and the diagonal form state space representations of the system.

8. An LTI system is characterised by the homogenous state equation

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}$$

(a)Find the state transition matrix and compute the solution of the equation assuming the initial vector

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(b)Consider now that the system has a forcing function and is represented by the following non-homogenous state equation:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

where u is unit step. Compute the solution of this equation assuming the initial conditions of part (a).

9. Consider a double-integrator plant defined by the differential equation $\frac{d^2\theta(t)}{dt^2} = u(t)$. Develop a state space model for the system with $\theta, \dot{\theta}$ as state variables x_1, x_2. Express the state equation in terms of variables $\bar{x}$ where $\bar{x}$ is the new state variable related to x by a similarity transformation $x = P\bar{x}$ and

$$P = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Show that the eigen values of the system matrices of the two representations of the system are equal.

10. Given the system

$$\dot{x} = Ax = \begin{bmatrix} -4 & 3 \\ -6 & 5 \end{bmatrix}$$

Determine the eigenvalues and eigenvectors of A, and use these results to find the state transition matrix.

11. Obtain state variable model in Jordan canonical form for the system with transfer function $\frac{2s^2+6s+5}{(s+1)^2(s+2)}$. Find the response to a unit step input using the state variable model obtained. Give a block diagram for analog computer simulation of the transfer function.

12. Using Cayley-Hamilton technique find e^{At} for

$$A = \begin{bmatrix} 0 & -3 \\ 1 & -4 \end{bmatrix}; A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$$

13. An LTI system is described by

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & 11 & -6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u; \; y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

Diagonalise the state matrix using a similarity transformation, and from there obtain explicit solutions of the state vector and output when u is unit step and

$$x(0) = \begin{bmatrix} 0 & 0 & 2 \end{bmatrix}^T$$

14. Consider the system given by

$$\dot{x} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 2 & 0 \\ 0 & 1 \end{bmatrix} u; \; y = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 0 \end{bmatrix} x$$

Comment about the controllability and observability of the system.

15. Consider the system

$$\dot{x} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} u; \; y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

Find the eigenvalues of A and then determine stability of the system. Find the transfer function model and from it determine stability. Why are the two conclusions about stability different?